Temporal Multiplexing Gains

Ikhlaq Sidhu, U.S.Robotics, Corporate Systems Division,
8100 N. McCormick Blvd, Skokie, IL 60076-2999, isidhu@usr.com

Hong Jiang, AT&T Bell Labs, Performance Analysis Department,
101 Crawfords Corner Rd, Holmdel, NJ 07733

Scott Jordan, Deptartment of Electrical Engineering and Computer Science,
Northwestern University, 2145 Sheridan Rd., Evanston, IL 60208-3118, scott@eecs.nwu.edu

Abstract

Effective Bandwidth is a powerful tool in modeling bandwidth and buffer requirements for multiplexed data sources. It is often approximated that an additional service capacity equal to the effective bandwidth of the source is required for every additional data source added to the multiplexed stream given a fixed buffer size and maximum blocking probability. This implies that a multiplexing gain can be achieved since additional buffer is not required with additional data sources. In this work, we consider other choices of bandwidth and buffer for a multiplexed case. By setting a price for bandwidth and buffer, we determine an optimal choice for the two resources. We consider the savings between simpler resource allocation schemes with that of the more complex optimal allocation scheme. We are interested to characterize the multiplexing gain so that we may determine if the savings in multiplexing outweighs the additional complexity of choosing optimal buffer and bandwidth at the multiplexor. In this regard, we find that the cost savings of optimal allocation over certain other allocation schemes is increasing and convex. By considering the case of On/Off Markov Fluids, we are able to obtain expressions for the optimal resource allocation path and resulting cost savings.

I. Introduction

In this paper, we consider a single homogenous service, multiplexing node with resources of buffer and bandwidth and assume costs for the respective resources. Based on the concept of effective bandwidth [1] [2] [3] [4] [5], the multiplexing node's resource requirements may be characterized by a bandwidth vs. buffer curve specific to the source type, number of sources, and QoS. Although, any combination of buffer and bandwidth on the curve will satisfy the QoS requirement, there is a unique, optimal operating point on the curve which minimizes the combined buffer and bandwidth costs.

We characterize the savings of optimal allocation over that using simpler allocation schemes parameterized by the number of sources being multiplexed. The three alternative schemes are Fixed Buffer, Incremented Buffer, and No Sharing. With Fixed Buffer (FB), the bandwidth resource increases by the 'effective bandwidth' with every source being multiplexed. With Incremented Buffer (IB), the buffer increases in fixed increments with the number of sources being multiplexed and the bandwidth is increased as necessary to satisfy the loss probability requirement. Finally, the No Sharing (NS) scheme considers the case when buffer and bandwidth are not shared between sources. We are interested to know if the cost benefit of optimal allocation outweighs the additional complexity. Additionally, we are interested to characterize the optimal allocation and cost savings with varying numbers of sources.

In section II, we define the model for multiplexing of homogenous sources using the schemes of optimal allocation, Fixed buffer allocation, Incremented Buffer allocation, and No Sharing allocation. This model relies on previous approximations that effective bandwidths are additive. More recent work has suggested improvements to that approximation. It is generally accepted that the additive nature of effective bandwidth is due to temporal gains of multiplexing while gains of capacity sharing are reflected in an effective bandwidth per source which decreases with the number of sources being multiplexed [6] [7]. In section III, we examine results for a wide variety of sources whose effective bandwidths are convex decreasing functions of buffer length. We give conditions for optimal allocation and proceed to show that the cost savings of optimal allocation over other allocation schemes is increasing and convex in the number of sources. In section IV, we consider Markov fluid On/Off sources and examine the multiplexing model and optimal resource allocation path as a function of number of sources.

II. Resource Allocation Model for Homogenous Multiplexing

In this section, we define a model to evaluate combinations of buffer and bandwidth at a multiplexor for differing numbers of homogenous sources given a constraint on blocking probability. Our model is based on the well-accepted concepts of effective bandwidth. By using a fixed cost for additional unit bandwidth and additional unit buffer we define an optimum resource allocation combination which varies with the number of sources being multiplexed. Further, we will compare the cost of the optimum allocation point with the cost of other allocation points obtained using simpler allocation schemes. We define the differences in the costs between these alternative schemes as cost savings which may be achieved through multiplexing.

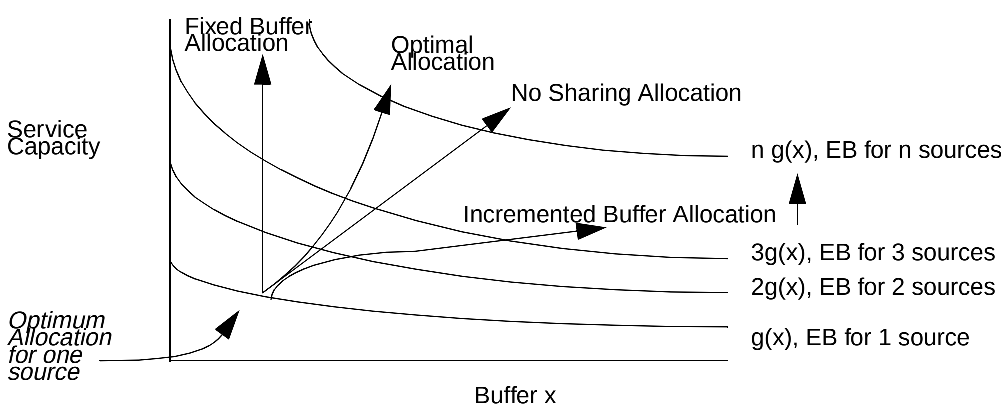

Figure 1 Resource allocation model

Consider a model to evaluate alternative buffer and bandwidth combinations parameterized by the number of sources being multiplexed (Figure 1). The curve labeled g(x) illustrates the effective bandwidth for one source as a function of buffer length x given a predefined loss constraint. We assume g(x) ranges between the peak rate and mean rate and is convex and decreasing in x [Elwa93]. Approximating the effective bandwidth of *n* sources into a buffer of length *x* as *n* times the effective bandwidth a single source into a buffer of length *x*, the corresponding curve for n sources is *n g(x)*.

To evaluate which allocation points may be more desirable, we define C_B and C_C to be the costs per unit buffer and bandwidth respectively and *m* to be the ratio C_B/C_C. We equate optimal resource allocation for *n* multiplexed sources with the least cost point on the n source effective bandwidth curve *n g(x)*. We are interested to characterize the path of optimal allocation points parameterized by *n*. The optimal cost $C^*(n)$ (EQ 3) can be obtained by the dot product of the cost vector (C_B,C_C) and the allocation point given in (EQ 2). The buffer value $x^*(n)$ which minimizes the system cost is also varying with the number of sources *n.* Define the optimal allocation point $P^*(n)$ In Figure 1, we illustrate a sample optimal allocation path.

$$x^*(n) = arg_x min \begin{bmatrix} C_B \\ C_C \end{bmatrix} \bullet \begin{bmatrix} x & ng(x) \end{bmatrix} \quad \text{(EQ 1)}$$

$$P^*(n) = [x^*(n), ng(x^*(n))] \quad \text{(EQ 2)}$$

$$C^*(n) = \begin{bmatrix} C_B \\ C_C \end{bmatrix} \bullet [x^*(n), ng(x^*(n))] \quad \text{(EQ 3)}$$

Consider an allocation scheme in which buffer is fixed at $x^*(1)$ and bandwidth is set to $n\, g(x^*(1))$. In this scheme, the allocation point for one source is optimal and an additional service capacity equal to the effective bandwidth is required for every additional source. We define allocation points for this scheme of *Fixed Buffer* $P_{FB}(n)$. The path of $P_{FB}(n)$ is a vertical line (Figure 1). The cost path of $P_{FB}(n)$ is given by $C_{FB}(n)$

$$P_{FB}(n) = [x^*(1), ng(x^*(1))] \quad \text{(EQ 4)}$$

$$C_{FB}(n) = C^*(1) + (n-1)g(x^*(1))C_C \quad \text{(EQ 5)}$$

Alternatively, we might increment the buffer resources by x*(1) with every source and choose the corresponding capacity $n\, g(n\, x^*(1))$ which takes advantage of multiplexing gains. This scheme of Incremented Buffer is given by (EQ 6) and (EQ 7) below.

$$P_{IB}(n) = [nx^*(1), ng(nx^*(1))] \quad \text{(EQ 6)}$$

$$C_{IB}(n) = \begin{bmatrix} C_B \\ C_C \end{bmatrix} \bullet [nx^*(1), ng(nx^*(1))] \quad \text{(EQ 7)}$$

Finally, consider the allocation scheme consistent with no sharing of resources. The allocation points are then given by $P_{NS}(n) = (nx^*(1), n\, g(x^*(1))$ and the path constitutes a line with slope $g(x^*(1))/x^*(1)$ (Figure 1). The cost path $C_{NS}(n)$ increases linearly with *n*.

$$P_{NS}(n) = [nx^*(1), ng(x^*(1))] \quad \text{(EQ 8)}$$

$$C_{NS}(n) = nC^*(1) = nC_{FB}(1) \quad \text{(EQ 9)}$$

Since $C^*(n)$ is defined by the minimum cost allocation, then fixed buffer scheme cost $C_{FB}(n)$, the incremented buffer cost scheme $C_{IB}(n)$, and the no sharing cost $C_{NS}(n)$ must each be greater in value. We define a cost saving $\Delta C^*_{FB}(n) = C_{FB}(n)-C^*(n)$ relating optimal allocation over fixed buffer allocation, a cost savings $\Delta C^*_{IB}(n) = C_{IB}(n)-C^*(n)$ of optimal allocation over the incremented buffer and a cost savings $\Delta C^*_{NS}(n) = C_{NS}(n)-C^*(n)$ of optimal allocation over no sharing.

Partial Variable List

- n: Number of homogenous sources being multiplexed
- $g(x)$: Effective bandwidth for a data source as a function of buffer length x.
- C_B: Cost per unit buffer.
- C_C: Cost per unit bandwidth or service rate.
- m: Ratio of C_B / C_C.
- $P^*(n)$, $P_{FB}(n)$, $P_{IB}(n)$, $P_{NS}(n)$: Optimal Allocation, Fixed Buffer, Incremented Buffer, and No Share buffer and bandwidth allocation paths for n multiplexed sources.
- $C^*(n)$, $C_{FB}(n)$, $C_{IB}(n)$, $C_{NS}(n)$: Cost of Optimal Allocation, Fixed Buffer, Incremented Buffer, and No Share combination.
- $x^*(n)$: Optimum buffer allocation for n multiplexed sources.
- $n^*(x)$: The inverse function of $x^*(n)$.
- $\Delta C^*_{FB}(n)$, $\Delta C^*_{IB}(n)$, $\Delta C^*_{NS}(n)$: Additional cost of $C_{FB}(n)$, $C_{IB}(n)$, $C_{NS}(n)$ and over $C^*(n)$.

III. General Sources

In this section, we are concerned with results for the wide category of data sources for which effective bandwidth functions are well behaved convex, decreasing functions of buffer given a maximum loss criteria. We start by examining the conditions of optimal allocation assignment. We will show that the optimal buffer allocation $x^*(n)$ is strictly increasing with the number of sources n. We obtain an expression for the relationship of bandwidth to buffer when optimal allocation is achieved and we will show that this relationship can be written as a product of the cost ratio m and a function of the effective bandwidth $g(x)$. We will finish this section by showing that the cost savings, $\Delta C^*_{FB}(n)$ and $\Delta C^*_{NS}(n)$, are both convex and increasing with the number of sources n.

Consider the effective bandwidth function $g(x)$ in Figure 2. We expect $g(0)$ to be equal or close to the peak rate of the source and $g(x)$ to approach the mean rate as x approaches infinity. We impose a broad assumption that $g(x)$ is a continuous, decreasing function of x, $x \in [0, \infty)$.

By (EQ 1), it follows that the equal cost line is tangent to the $n\, g(x^*(n))$ curve as in (EQ 10). If this condition can not be met, then $x^*(n) = 0$.

$$n \cdot \frac{dg}{dx^*} = -m \qquad x^*(n) > 0 \tag{EQ 10}$$

For non-zero *x*, we may reverse (EQ 10) to obtain the number of sources which can be accommodated under the optimality condition. In this analysis, we allow *n*(x)* to be real valued.

$$n^*(x) = \frac{-m}{\frac{d}{dx}g(x)} \qquad x > 0 \tag{EQ 11}$$

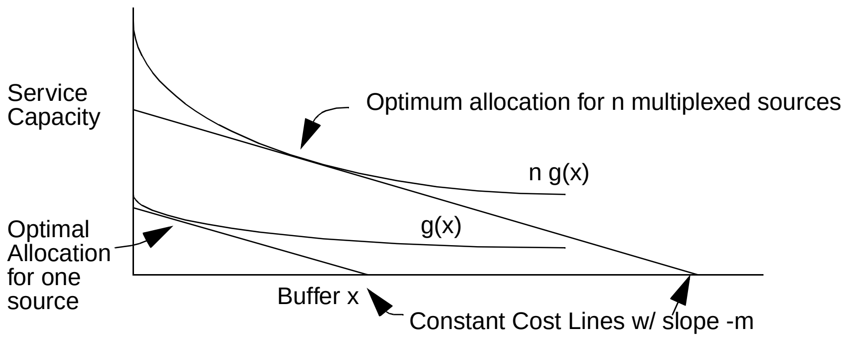

Figure 2 Minimum cost conditions

We now show that the optimal buffer *x*(n)* is strictly increasing with n in the range where *x*(n)* >0.

Theorem 1:

An optimal buffer assignment *x*(n)* which solves the minimization problem in (EQ 1) is strictly increasing with the number of sources *n* for *x*(n)* >0.

proof:

Suppose $x^*(n) \ge x^*(n+1)$ then

$$n\frac{d}{dx}g(x^*(n)) = (n+1)\frac{d}{dx}g(x^*(n+1)) = -m$$

$$\frac{\frac{d}{dx}g(x^*(n+1))}{\frac{d}{dx}g(x^*(n))} = \frac{n}{n+1} < 1$$

However, if $x^*(n) \ge x^*(n+1)$ then

$$\frac{\frac{d}{dx}g(x^*(n+1))}{\frac{d}{dx}g(x^*(n))} \ge 1$$

since *dg(x)/dx* is negative and strictly decreasing in magnitude. The contradiction implies x*(n) < x*(n+1).

Next, we consider the path of optimal buffer and bandwidth allocation *P*(n)*. By combining (EQ 2) and (EQ 11), we are able to eliminate the parametric variable *n* and show the relationship of bandwidth to buffer. Note that the capacity resource for a given buffer *x* is the ratio of the effective bandwidth to the magnitude of its derivative scaled by the cost ratio *m.*

$$P^*(n(x)) = \left[x(n), \frac{-m}{\frac{dg}{dx}} g(x(n)) \right] = \left[x, \frac{mg(x)}{\left|\frac{dg}{dx}\right|} \right] \qquad \forall x \geq x^*(1) \qquad \text{(EQ 12)}$$

Figure 3 illustrates an optimal path. From theorem 1, the number of sources *n* and the buffer *x* increase together. The valid domain of the optimal path *P*(n*(x))* is illustrated to depend on the boundary condition *x*(1)*. It can be shown that if *g(x)* has an algebraic, asymptotic form $g(x) = B + Ax^{-\nu}, \nu \in (0, \infty)$, then *P*(n(x))* will be convex and increasing.

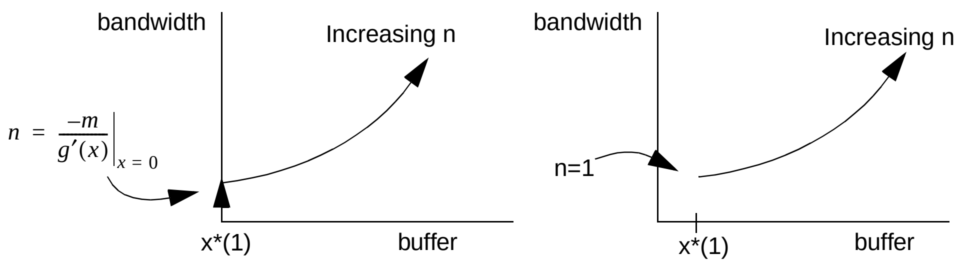

Figure 3 Boundary conditions on optimal path of buffer and bandwidth allocation.

The optimal allocation paths above show how resources increase along the path of minimum cost *P*(n)*. We compare this minimum cost path with the cost paths of the fixed buffer allocation and the no sharing allocation. In the following 2 theorems, we show the cost savings to be increasing convex with *n* .

Theorem 2:

The cost saving $\Delta C^*_{FB}(n)$ of optimal allocation over fixed buffer allocation is i) increasing with *n* and ii) convex in n for n such that *x*(n)*>0.

proof:

Figure 4 illustrates the effective bandwidth curves for *n, n+1, and n+2* multiplexed sources. The points labeled *P*(n), P*(n+1),* and *P*(n+2)* are optimal resource allocation points for *n, n+1,* and *n+2* sources respectively and the slope of the curve tangent to each of these points is identically *-m* by the optimality condition. The tangent lines shown may be considered as constant cost lines. The distances labeled Δ_n, Δ_{n+1}, and Δ_{n+2} are proportional to the extra cost of the *Fixed Buffer* scheme over the optimal allocation points respectively since each point on the tangent lines shown have equal costs. We recognize that the additional cost $\Delta C^*_{FB}(n)$ is equal to $C_C \Delta_n$. Formulate Δ_n, Δ_{n+1}, and Δ_{n+2} as follows:

$$\Delta_n = \int_{x^*(1)}^{x^*(n)} \left[n\left[-\frac{\partial}{\partial x}(g(x))\right] - m\right]dx = \int_{x^*(1)}^{x^*(n)} n|g'(x)| - mdx$$

$$\Delta_{n+1} = \int_{x^*(1)}^{x^*(n)} (n+1)|g'(x)| - mdx + \int_{x^*(n)}^{x^*(n+1)} (n+1)|g'(x)| - mdx$$

$$\Delta_{n+2} = \int_{x^*(1)}^{x^*(n)} (n+2)|g'(x)| - mdx + \int_{x^*(n)}^{x^*(n+1)} (n+2)|g'(x)| - mdx + \int_{x^*(n+1)}^{x^*(n+2)} (n+2)|g'(x)| - mdx$$

To show that $\Delta C^*_{FB}(n)$ is strictly increasing, we simply take the difference of Δ_{n+1}-Δ_n. The result can be shown to be the integral of a positive quantity over the positive distance from *x*(n)* to *x*(n+1)* (from Theorem 1) and a second positive quantity integrated over *x*(1)* to *x*(n)*.

Let us now prove the convexity of $\Delta C^*_{FB}(n)$ by showing that (Δ_{n+2}-Δ_{n+1})-(Δ_{n+1}-Δ_n) is positive:

$$(\Delta_{n+2} - \Delta_{n+1}) - (\Delta_{n+1} - \Delta_n) = \int_{x^*(n)}^{x^*(n+1)} m - n|g'(x)|dx + \int_{x^*(n+1)}^{x^*(n+2)} (n+2)|g'(x)| - mdx \qquad \text{(EQ 13)}$$

If both integrals are positive then the sum of the two is indeed positive. Let us eliminate *m* from the above integrals by using the optimality condition from (EQ 10) as specified below:

$$\begin{aligned} m &= n|g'(x^*(n))| \\ m &= (n+2)|g'(x^*(n+2))| \end{aligned} \qquad \text{(EQ 14)}$$

The first integral takes the following form:

$$\int_{x^*(n)}^{x^*(n+1)} m - n|g'(x)|dx = n \int_{x^*(n)}^{x^*(n+1)} |g'(x^*(n))| - |g'(x)|dx$$

In the range of *x* between *x*(n)* to *x*(n+1)*, the value of the integrated quantity is positive since the magnitude of the derivative $|g'(x)|$ is decreasing in value by the convex, decreasing assumption on *g(x)* and Theorem 1. Hence, the integral is positive.

The second integral in (EQ 13) is also found to be positive by substitution of the second line in (EQ 14):

$$\int_{x^*(n+1)}^{x^*(n+2)} (n+2)|g'(x)| - mdx = (n+2) \int_{x^*(n+1)}^{x^*(n+2)} |g'(x)| - |g'(x^*(n+2))|dx$$

In this case $|g'(x)|$ is greater or equal with $|g'(x^*(n+2))|$ in the range of *x* between *x*(n+1)* and *x*(n+2)* by Theorem 1 and the assumed convexity of *g(x)*.

Therefore (Δ_{n+2}-Δ_{n+1})-(Δ_{n+1}-Δ_n) is positive for *n* which satisfy the optimality conditions, i.e. *x*(n)>0* and $n \geq 1$.

Theorem 3:

The cost saving $\Delta C^*_{NS}(n)$ of optimal allocation over no sharing allocation is i) increasing with n and ii) convex in n for n such that $x^*(n)>0$.

proof:

From (EQ 4) and (EQ 8), we see that the additional resource of no sharing over fixed buffer allocation is $(n-1)x^*(1)$ with a linearly scaling cost of $C_B(n-1)x^*(1)$. Thus, our quantity of interest, $\Delta C^*_{NS}(n) = \Delta C^*_{FB}(n) + C_B(n-1)x^*(1)$. $\Delta C^*_{FB}(n)$ is increasing and convex from Theorem 2. The additive term $C_B(n-1)x^*(1)$ is linearly increasing with n. Therefore, $\Delta C^*_{NS}(n)$ is also increasing in and convex in n with the same restrictions for n as given in Theorem 2.

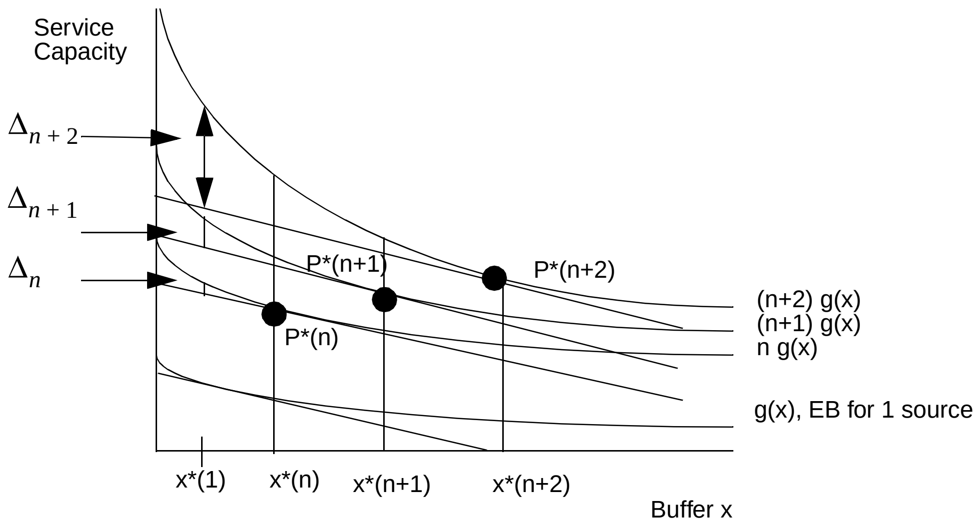

Figure 4 Illustration of cost savings of optimal allocation over fixed bandwidth allocation.

IV. On/Off Markov Fluids Sources

In this section, we proceed to give expressions for quantities of interest including optimal path $P^*(n(x))$, optimal cost path $C^*(n)$, and cost savings $\Delta C^*_{FB}(n)$, $\Delta C^*_{IB}(n)$, and $\Delta C^*_{NS}(n)$ specific to the case of On/Off Markov fluid sources.

Consider this alternate expression for effective bandwidth $g_{On/Off}(x)$ in terms of buffer length where λ is the on rate [Elwa93]:

$$g_{\mathrm{On/Off}}(x) = a - bx + \sqrt{b^2x^2 + cx + a^2} \qquad a = \frac{\lambda}{2} \qquad b = \frac{\alpha+\beta}{2\gamma} \qquad c = \frac{(\alpha-\beta)\lambda}{2\gamma}$$
$$Generator = \begin{bmatrix} -\alpha & \alpha \\ \beta & -\beta \end{bmatrix} \qquad Rate = \begin{bmatrix} 0 \\ \lambda \end{bmatrix} \qquad \gamma = |ln(pb)| \tag{EQ 15}$$

Note, the effective bandwidth with no buffer is simply $g_{On/Off}(0) = 2a = \lambda$. The limiting value of $g(x)$ as $x \to \infty$ is the mean rate of the source, $\lambda\left(\frac{\alpha}{\alpha+\beta}\right)$.

The optimal path for the On/Off source case is then found by substituting (EQ 15) into *P*(n(x))* in (EQ 12).

$$P^*_{\text{On/Off}}(n(x)) = \left[x, m\frac{(a - bx + \sqrt{b^2x^2 + cx + a^2})}{\left|\frac{b^2x + c/2}{\sqrt{b^2x^2 + cx + a^2}} - b\right|}\right] \quad \text{(EQ 16)}$$

Now, we introduce a Taylor Series approximation for the effective bandwidth function so that we might characterize the nature of the optimal path, the multiplexing gains, and their respective sensitivities. Consider the following Taylor Series expansion of $g_{ON/Off}(x)$:

$$g_{\text{On/Off}}(x) = k_0 + \frac{k_1}{x} + \frac{k_2}{x^2} + O(x^{-3})$$
$$k_0 = \frac{\lambda\alpha}{\alpha + \beta} \qquad k_1 = \frac{\lambda^2\gamma}{4(\alpha+\beta)}\left[1 - \left(\frac{\alpha-\beta}{\alpha+\beta}\right)^2\right] \qquad k_2 = -\lambda^3\gamma^2\frac{(\alpha-\beta)}{4(\alpha+\beta)^3}\left[1 - \left(\frac{\alpha-\beta}{\alpha+\beta}\right)^2\right] \quad \text{(EQ 17)}$$

We find the effective bandwidth to be decreasing to the mean rate k_0 with a dominant factor of k_1/x as x grows to infinity. It can be shown that k_0 and k_1 are both positive quantities. Using this approximation, $P^*(n(x))_{On/Off}$, can be expressed as:

$$P^*_{\text{On/Off}}(n(x)) = \left|x(n), \frac{-m}{\frac{dg}{dx}}g(x(n))\right| = \left[x, m\frac{k_0}{k_1}x^2 + O(x)\right] \quad \text{(EQ 18)}$$

We see that the capacity requirement is increasing with the square of buffer length which is consistent with illustration of a convex, increasing optimal allocation path (Figure 1).

The cost savings $\Delta C^*_{FB,\text{ On/Off}}(n)$, $\Delta C^*_{IB,\text{ On/Off}}(n)$, and $\Delta C^*_{NS,\text{ On/Off}}(n)$ of optimal allocation over Fixed Buffer allocation, Incremented Buffer allocation, and no sharing respectively for the case of On/Off Markov fluid sources can be found from (EQ 2) - (EQ 9), (EQ 17), and (EQ 18).

$$\Delta C^*_{FB,\text{ On/Off}}(n) = nC_C[g(x^*(1)) - k_0] - 2\sqrt{n}\sqrt{C_BC_Ck_1} + O(1) \quad \text{(EQ 19)}$$

$$\Delta C^*_{IB,\text{ On/Off}}(n) = nC_B(x^*(1)) - 2\sqrt{n}\sqrt{C_BC_Ck_1} + O(1) \quad \text{(EQ 20)}$$

$$\Delta C^*_{NS,\text{ On/Off}}(n) = n[C_C(g(x^*(1)) - k_0) + C_Bx^*(1)] - 2\sqrt{n}\sqrt{C_BC_Ck_1} + O(1) \quad \text{(EQ 21)}$$

Each of the multiplexing gains have a form of $c_0n - c_1\sqrt{n} + O(1)$ and agree with the previous results concerning convexity of cost savings. The savings over the Fixed Buffer scheme is increasing with a dominant term of *n* times *(g(x*(1))* minus the mean source rate scaled by the bandwidth cost C_C. The savings over the Incremented Buffer scheme has a dominant term of *n* times the optimal buffer value *x*(1)* scaled by the buffer cost C_B. Finally, the savings over the No Sharing Scheme shows its dominant value term to be the sum of the previous two dominant term values.

V. Summary

In this work, we have introduced the concept of an increasing, convex optimal resource path of buffer and bandwidth parameterized by the number of homogenous sources being multiplexed. The gains of choosing optimal resource allocations due to temporal gains in multiplexing have been shown to be convex, increasing functions of the number of sources being multiplexed over certain alternative allocation schemes. For Markov Fluid On/Off sources, the cost savings is shown to have a form of $c_0 n - c_1 \sqrt{n} + O(1)$. This work is applicable to situations where resources in a switch or multiplexing system must be partitioned among different source classes. A study of this phenomena which incorporates the non-temporal gains in multiplexing would be a natural extension and future direction.

VI. References

[1] A. I. Elwalid and D. Mitra, “Analysis and design of rate based congestion control of high speed networks, I: stochastic fluid models, access regulations,” *Queueing Systems* 9 pp. 29-64, 1991.

[2] A. I. Elwalid, D. Mitra, “Effective Bandwidths of General Markovian Traffic Sources and Admission Control of High Speed Networks”, *IEEE/ACM Transactions in Networking, Vol. 1, No. 3,* pp. 329-343, 1993.

[3] R.J. Gibbens and P.J. Hunt, “Effective bandwidths for the multi-type UAS channel,” *Queueing Systems* 9 pp. 17-28, 1991.

[4] Roch Guerin, Hamid Ahmadi, and Mahmoud Naghshineh, “Equivalent Capacity and Its Application to Bandwidth Allocation in High Speed Network,” *IEEE JSAC* Vol. 9, No. 7, pp. 968-981, 1991.

[5] G. Kesidis, J. Walrand, and C.-S. Chang, “Effective Bandwidths for Multiclass Markov Fluids and Other ATM Sources, *IEEE ACM / Transactions on Networking*, Vol. 1, No. 4. pp. 424-428, August 1993.

[6] K. Rege, “Equivalent Bandwidth and Related Admission Criteria for ATM systems - A Performance Study,” *International Journal of Analog and Digital Communication Systems*, Vol 7, July, pp.181-191, 1994.

[7] I. Sidhu and S. Jordan, “Multiplexing Gains in Bitstream Multiplexors,” To appear in *IEEE / Transactions on Networking.*